Summary of Unit 4

- The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.
- The medians of a triangle are concurrent.
- The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base or in the ratio of 2 : 1 from the vertex.
- The point which divides the median in a triangle in the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.
- In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.
- If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.
- The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.
- The base angles of the isosceles triangle are congruent. (i.e. equal in measure)
- If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.
- If the triangle is equilateral , then it is equiangular where each angle measure is 60°
- If the angles of a triangle are congruent , then the triangle is equilateral.
- The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.
- The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.
- The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

- The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.
- The axis of symmetry of a line segment is the straight line perpendicular to it from its midpoint.
- Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).
- If a point is at equal distances from the two terminals of a line segment, then this point lies on the axis of this line segment.
- The isosceles triangle has one axis of symmetry which is the straight line perpendicular from its vertex to its base.
- The equilateral triangle has three axes of symmetry.
- The scalene triangle has no axes of symmetry.

Exams on Unit Four

Model 1

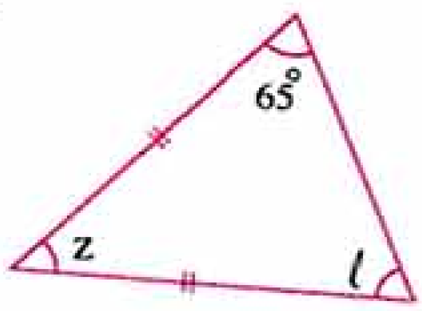
Answer the following questions :

1 Choose the correct answer from those given :

1. If M is the point of intersection of the medians in Δ ABC and $\overline{AD}$ is a median of length 6 cm. , then AM = ...
   (a) 1 cm. (b) 4 cm. (c) 3 cm. (d) 2 cm.
2. If the measure of a base angle of an isosceles triangle is 40° , then the measure of the vertex angle is ....
   (a) 40° (b) 50° (c) 80° (d) 100°
3. The measure of the exterior angle of the equilateral triangle equals ....
   (a) 30° (b) 60° (c) 90° (d) 120°
4. If the point A lies on the axis of symmetry of $\overline{XY}$, then $\overline{AX}$ .... $\overline{AY}$
   (a) // (b) $\perp$ (c) $\equiv$ (d) =
5. If ABC is a right-angled triangle at A and AB = AC , then m ($\angle$ B) = ...
   (a) 30° (b) 45° (c) 60° (d) 90°
6. The number of axes of symmetry of the isosceles triangle is ....
   (a) 0 (b) 1 (c) 2 (d) 3

2 Complete the following :

1. The point of intersection of the medians of the triangle divides each of them in the ratio .... : 2 from the vertex.
2. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....
3. The median of the isosceles triangle drawn from the vertex .... , ...
4. If the length of the median of the triangle which is drawn from one of its vertices equals half the length of the opposite side to this vertex , then ....
5. **In the opposite figure :**
   ℓ = ...°
   z = ...°

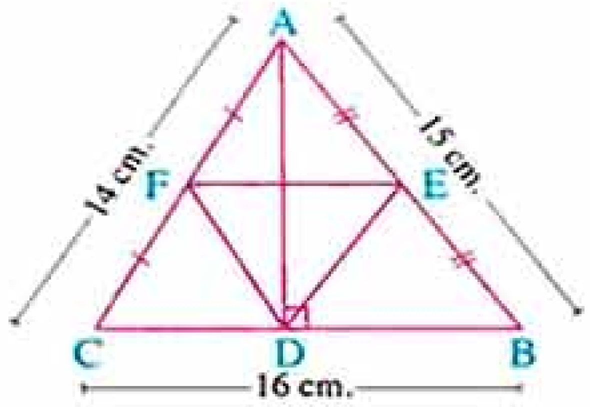


3 **[a] In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, E is the midpoint of $\overline{AB}$

and F is the midpoint of $\overline{AC}$

Find : The perimeter of Δ DEF

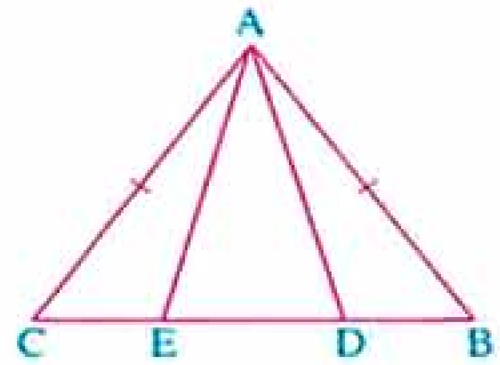


[b] In the opposite figure :

m ($\angle$ BAE) = m ($\angle$ CAD)

and AB = AC

Prove that : AE = AD



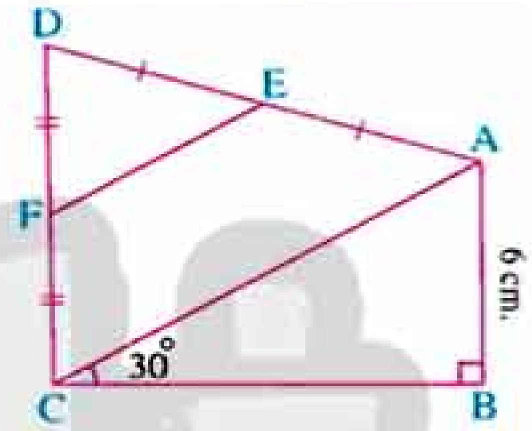
4 **[a] In the opposite figure :**

m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

AB = 6 cm. , E is the midpoint of $\overline{AD}$

and F is the midpoint of $\overline{DC}$

Find : The length of $\overline{EF}$

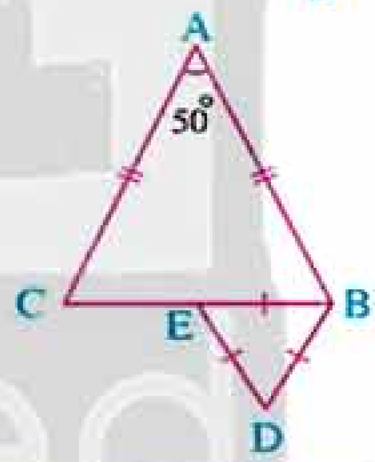


[b] In the opposite figure :

AB = AC , m ($\angle$ A) = 50°

and Δ BDE is an equilateral triangle.

Find : m ($\angle$ ABD)



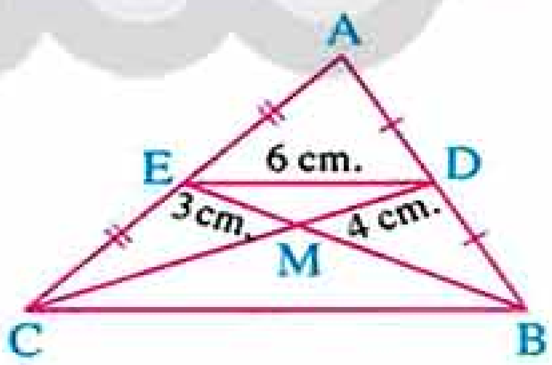
5 **[a] In the opposite figure :**

$\overline{BE}$ and $\overline{CD}$ are two medians of Δ ABC

intersecting at M , ME = 3 cm.

, MD = 4 cm. and DE = 6 cm.

Find : The perimeter of Δ MBC



[b] In the opposite figure :

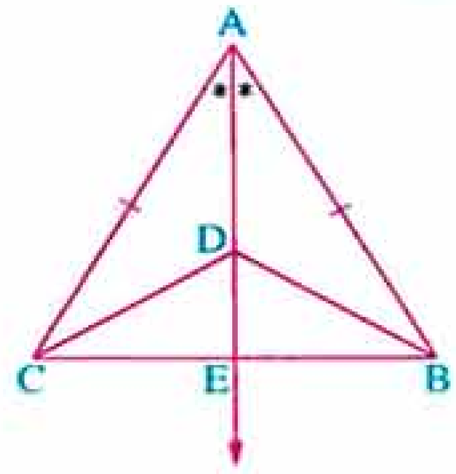
ABC is a triangle in which : AB = AC

, $\overrightarrow{AE}$ bisects $\angle$ BAC

, $\overrightarrow{AE} \cap \overline{BC} = \{E\}$ and D $\in \overrightarrow{AE}$

Prove that : **1** BE = $\frac{1}{2}$ BC

2 BD = CD



Unit 4

Model 2

Answer the following questions :

1 Choose the correct answer from those given :

1 The base angles of the isosceles triangle are ....

(a) complementary. (b) supplementary. (c) congruent. (d) straight.

2 If M is the point of intersection of the medians of Δ ABC , D is the midpoint of $\overline{BC}$, then AD = ...

(a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

3 If the measure of the vertex angle of an isosceles triangle is 50° , then the measure of each of the base angles is ....

(a) 40° (b) 65° (c) 70° (d) 130°

4 ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, then BD = ...

(a) $\frac{1}{2}$ AC (b) AC (c) $\frac{1}{2}$ BC (d) AB

5 The triangle which has three axes of symmetry is ....

(a) isosceles. (b) equilateral. (c) right-angled. (d) obtuse-angled.

6 In Δ ABC , if AB = AC , m ($\angle$ A) = 2 m ($\angle$ B) , then m ($\angle$ C) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

2 Complete the following :

1 The bisector of the vertex angle of an isosceles triangle is .... , ...

2 Any point on the axis of symmetry of a line segment is at .... distances from its two terminals.

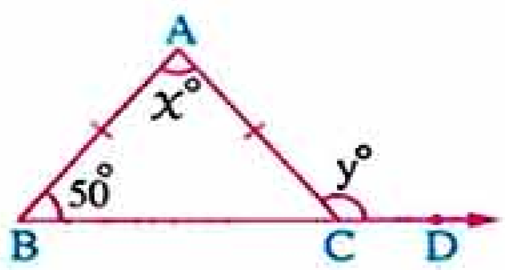
3 ABC is a right-angled triangle at B , m ($\angle$ C) = 30° , AB = 4 cm. , then AC = ... cm.

4 **In the opposite figure :**

AB = AC , D $\in \overrightarrow{BC}$

, then x = ...

, y = ...

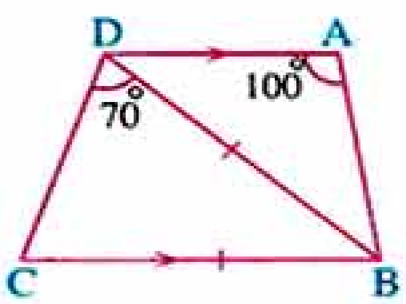


3 [a] In the opposite figure :

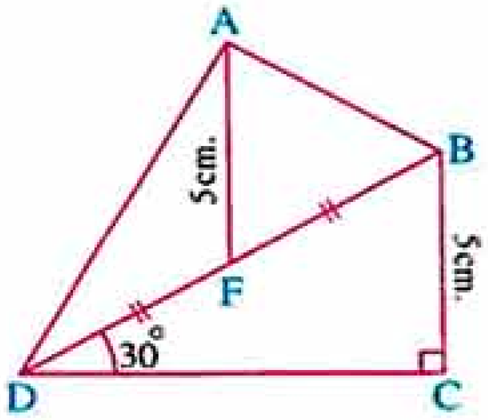
$\overline{AD}$ // $\overline{BC}$, BD = BC

, m ($\angle$ A) = 100° and m ($\angle$ BDC) = 70°

Prove that : Δ ABD is isosceles.



[b] In the opposite figure :



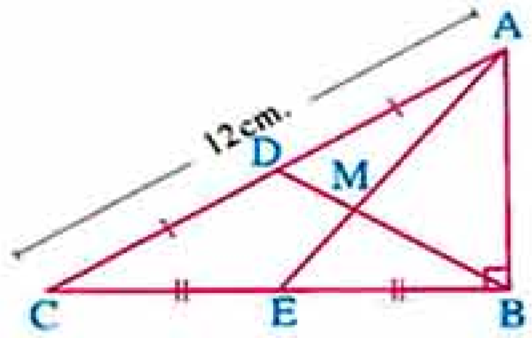
m (∠ C) = 90° , $\overline{AF}$ is a median in Δ ABD , m (∠ BDC) = 30°

and BC = AF = 5 cm.

1 **Find :** The length of $\overline{BD}$

2 **Prove that :** m (∠ BAD) = 90°

4 **[a] In the opposite figure :**



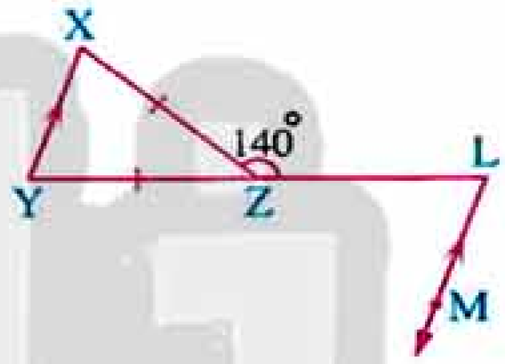
m (∠ ABC) = 90°

, AD = DC and BE = EC

If AC = 12 cm.

Find the length of each of : $\overline{BD}$ and $\overline{MD}$

[b] In the opposite figure :



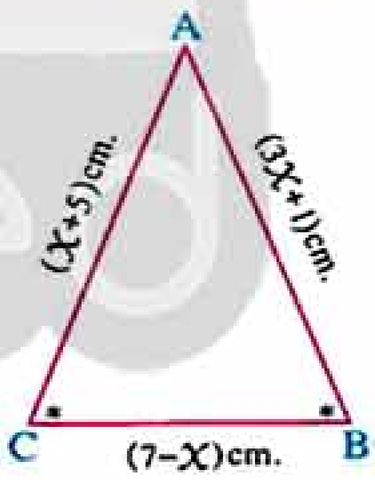
Z ∈ $\overline{LY}$, XZ = YZ

, m (∠ LZX) = 140°

and $\overrightarrow{LM}$ // $\overline{XY}$

Find : m (∠ MLY)

5 **[a] In the opposite figure :**

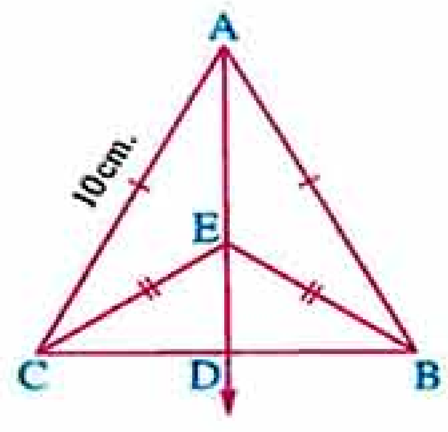


ABC is a triangle in which

m (∠ B) = m (∠ C)

Find : The perimeter of Δ ABC

[b] In the opposite figure :



AB = AC = 10 cm.

, EB = EC and $\overrightarrow{AE} \cap \overline{BC} = \{D\}$

1 **Prove that :** BD = DC

2 If BC = 6 cm. , **find the length of each of :** $\overline{CD}$ and $\overline{AD}$

Summary of Unit 5

- **Axioms of inequality relation :**

  For any four numbers a , b , c and d :

  1. If $a > b$, then $a + c > b + c$
  2. If $a > b$, then $a - c > b - c$
  3. If $a > b$, $c > 0$, then $a c > b c$
  4. If $a > b$, $b > c$, then $a > c$
  5. If $a > b$, $c > d$, then $a + c > b + d$

- In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the greater measure.
- In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.
- In the right-angled triangle , the hypotenuse is the longest side.
- The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.
- The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.
- **Triangle inequality :**

  In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.
- The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

Exams on Unit Five

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. The sum of lengths of any two sides of a triangle .... the length of the third side.
   (a) is smaller than (b) is greater than (c) equals (d) equals twice
2. In Δ ABC , if m ($\angle$ B) > m ($\angle$ C) , then ....
   (a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{BC}$
3. If the lengths of two sides in an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side equals ....
   (a) 7 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.
4. Which of the following numbers can be lengths of sides of a triangle ?
   (a) 2 , 3 , 4 (b) 2 , 3 , 5 (c) 2 , 3 , 6 (d) 2 , 3 , 7
5. In Δ ABC , if m ($\angle$ C) = 65° and m ($\angle$ A) = 75° , then ....
   (a) AB > BC (b) AB < AC (c) BC > AB (d) AB = AC
6. In Δ ABC , if m ($\angle$ B) = 130° , then its longest side is ....
   (a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

2 Complete the following :

1. If two sides in a triangle are unequal in length , then the longer of them is opposite to an angle ....
2. The longest side of the right-angled triangle is ....
3. In Δ ABC , if AB < BC < AC , then the smallest angle in measure is ....
4. **In the opposite figure :**
   If B , C belong to $\overleftrightarrow{AD}$, such that
   DC > AB , then AC .... DB

5. ABC is a triangle in which : AB = 5 cm. and BC = 3 cm. , then AC ∈] .... , ... [

Unit 5

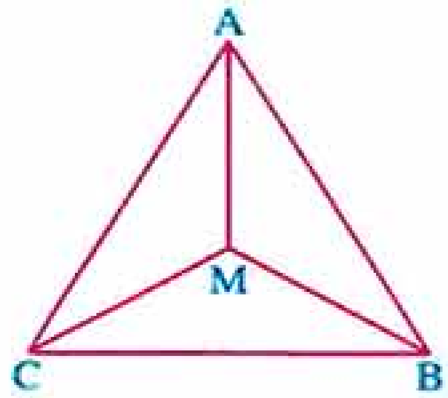
3 **[a]** In Δ ABC : m ($\angle$ A) = 30° and m ($\angle$ B) = 65°

Arrange the lengths of the sides of the triangle descendingly.

[b] ABCD is a quadrilateral in which : AB = 6 cm. , BC = 3 cm. , CD = 4 cm. and DA = 5 cm.

Prove that : m ($\angle$ DCB) > m ($\angle$ DAB)

4 **[a] In the opposite figure :**

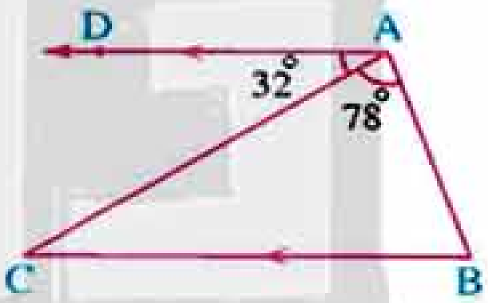


ABC is a triangle

and M is a point inside it.

Prove that : MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

[b] In the opposite figure :

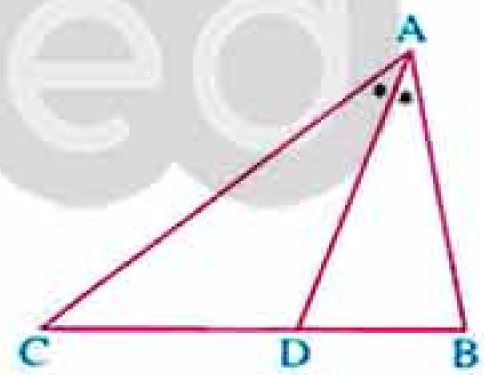


$\overrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 78°

and m ($\angle$ CAD) = 32°

Prove that : AC > AB

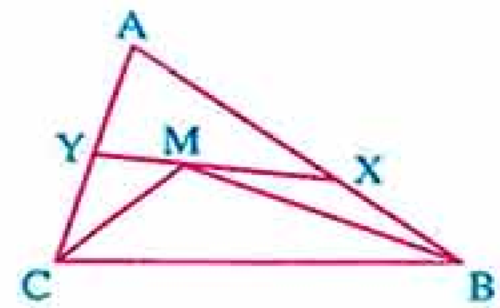
5 **[a] In the opposite figure :**



$\overrightarrow{AD}$ bisects $\angle$ A

Prove that : AC > DC

[b] In the opposite figure :



ABC is a triangle in which : X $\in \overline{AB}$

, Y $\in \overline{AC}$ and M $\in \overline{XY}$

Prove that : AB + AC > MB + MC

Model 2

Answer the following questions :

1 Choose the correct answer from those given :

1 If the triangle ABC is right-angled at B , then ....

(a) AC < AB (b) AC < BC (c) AB < AC (d) BC = AB

2 A triangle of two side lengths 4 cm. and 9 cm. , and it has one axis of symmetry , then the length of the third side equals ....

(a) 4 cm. (b) 5 cm. (c) 9 cm. (d) 13 cm.

3 The length of any side in a triangle .... the sum of lengths of the two other sides.

(a) is smaller than (b) is greater than (c) equals (d) is half

4 Δ ABD is an obtuse-angled triangle at B , C is the midpoint of $\overline{BD}$, then the greatest side in length is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BD}$ (d) $\overline{AD}$

5 Which of the following numbers can't be lengths of sides of a triangle ?

(a) 3 , 4 , 4 (b) 3 , 4 , 5 (c) 3 , 4 , 6 (d) 3 , 4 , 7

6 In Δ XYZ , XY + YZ – XZ ....

(a) > 0 (b) < 0

(c) = 0 (d) = the perimeter of Δ XYZ

2 Complete the following :

1 If two angles are unequal in measure in a triangle , then the greater angle in measure is opposite to ....

2 In the isosceles triangle ABC , if AB = AC , m ($\angle$ A) = 70° , then AB < ....

3 In the triangle ABC , if m ($\angle$ A) = 67° , m ($\angle$ B) = 33° , then AB > .... > ....

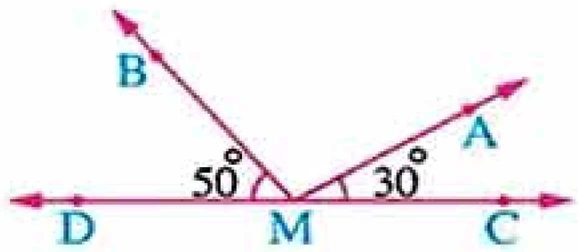
4 If ABC is a triangle in which m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then the greatest side in length is ....

Unit 5

5 In the opposite figure :

$M \in \overleftrightarrow{CD}$

, then m (∠ CMB) .... m (∠ AMD)

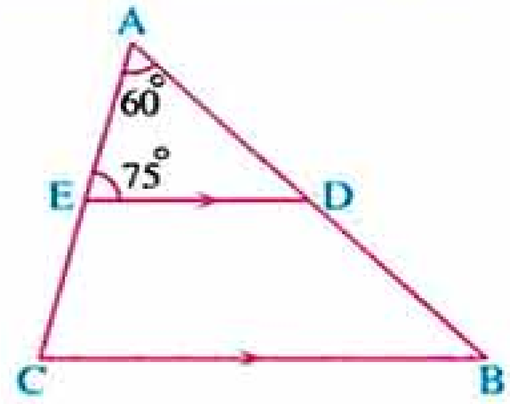


3 [a] In the opposite figure :

$\overline{ED} \parallel \overline{BC}$, m (∠ A) = 60°

and m (∠ AED) = 75°

Prove that : AB > AC

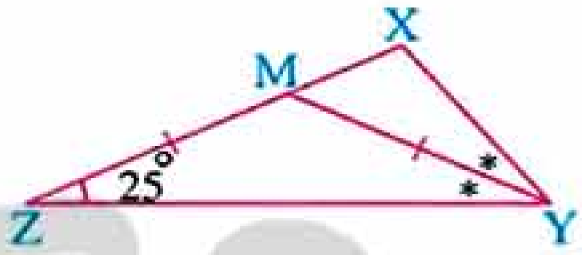


[b] In the opposite figure :

$\overrightarrow{YM}$ bisects ∠ XYZ

, MY = MZ and m (∠ Z) = 25°

Prove that : YM > XY



4 [a] ABC is a triangle in which AB = 7 cm.

, BC = 4 cm. and CA = 5 cm.

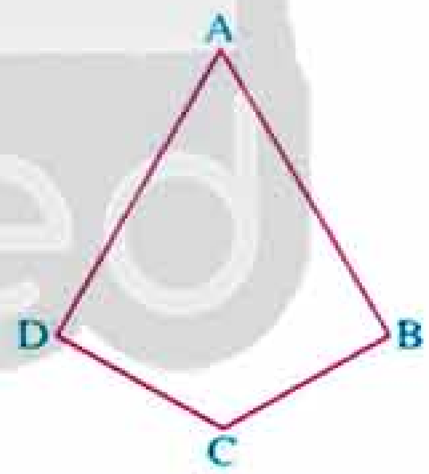
Arrange the angles of the triangle ascendingly due to their measures.

[b] In the opposite figure :

AB > BC

and AD > DC

Prove that : m (∠ BCD) > m (∠ BAD)



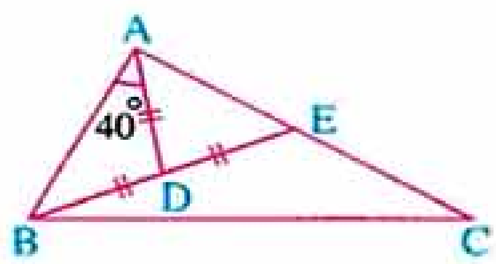
5 [a] In the opposite figure :

AD = BD = DE and m (∠ DAB) = 40°

Prove that :

1 AD < AB

2 BC > AC

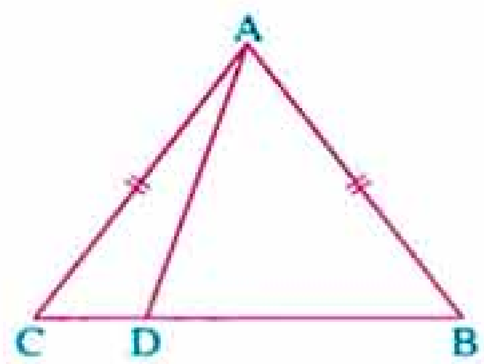


[b] In the opposite figure :

AB = AC

and $D \in \overline{BC}$

Prove that : AB > AD



Quiz 1 on lesson 1 – unit 4

time 20 min.

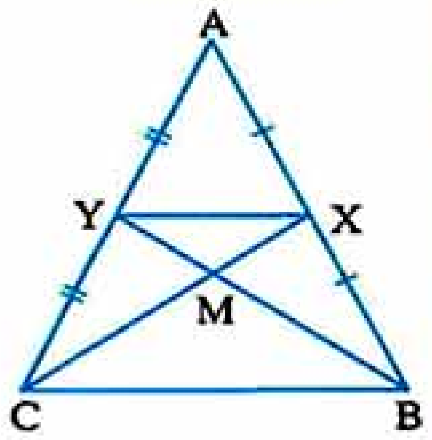
1 Complete the following :

1. The medians of the triangle intersect at ....
2. The point of intersection of the medians of the triangle divides each of them by the ratio .... : .... from the vertex.
3. If $\overline{AD}$ is a median in Δ ABC and M is the point of intersection of its medians , AM = 6 cm. , then AD = ... cm.

2 [a] In the opposite figure :

ABC is a triangle , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, XM = 4 cm. , XY = 5 cm. , BY = 12 cm.

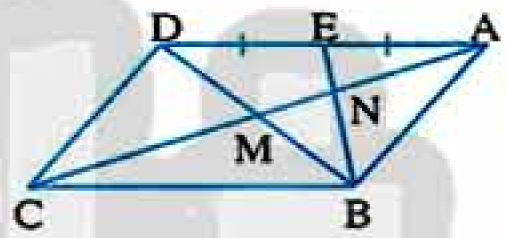
Find : The perimeter of Δ MBC



[b] In the opposite figure :

ABCD is a parallelogram whose diagonals intersect at M , E is the midpoint of $\overline{AD}$, $\overleftrightarrow{BE} \cap \overline{AC} = \{N\}$

Prove that : $AN = \frac{1}{3} AC$



Quiz 2 till lesson 2 – unit 4

time 20 min.

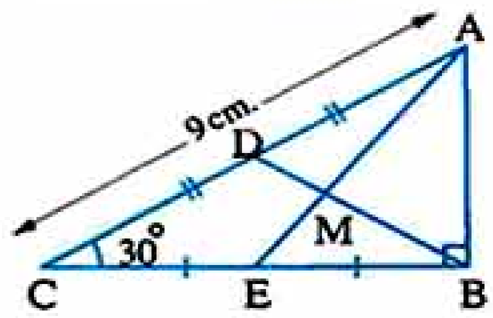
1 Complete the following :

1. The length of the median drawn from the vertex of the right angle of the right-angled triangle = ...
2. In Δ ABC if $\overline{AD}$ is a median of length 12 cm. , M is the point of intersection of medians , then AM = ... cm.
3. The length of the side opposite to the angle whose measure = 30° in the right-angled triangle = ...

2 [a] In the opposite figure :

ABC is a triangle in which : m ($\angle$ B) = 90° , m ($\angle$ C) = 30° , AC = 9 cm. , $\overline{AE}$ and $\overline{BD}$ are two medians intersecting at M

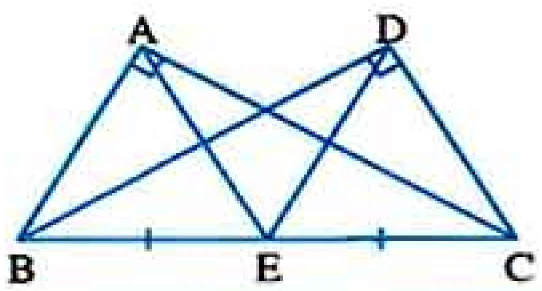
Find : The length of each of $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$



[b] In the opposite figure :

m ($\angle$ BAC) = m ($\angle$ BDC) = 90° , E is the midpoint of $\overline{BC}$

Prove that : AE = DE



Quiz 3 till lesson 3 – unit 4

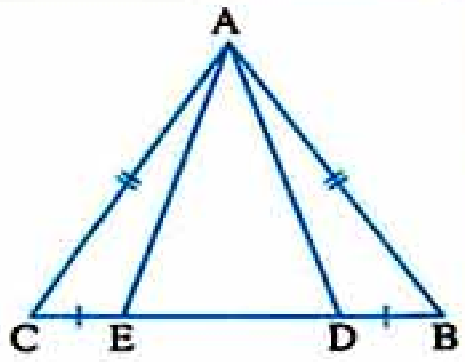
time 20 min.

1 Complete the following :

1. The measure of any exterior angle of the equilateral triangle = ……………°
2. ABC is an isosceles triangle in which AB = AC , m (∠ A) = 110° , then m (∠ B) = ……………°
3. If the length of the median which is drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is ……………

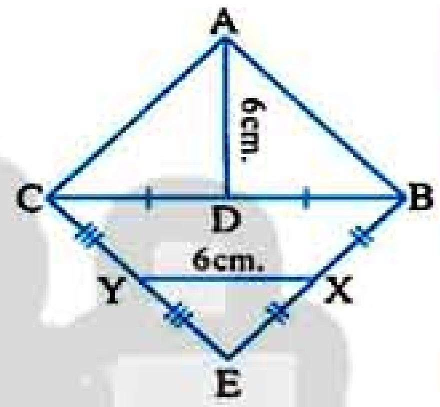
2 [a] In the opposite figure :

ABC is a triangle in which : AB = AC
, D ∈ $\overline{BC}$ and E ∈ $\overline{BC}$
such that : BD = EC
Prove that : AD = AE



[b] In the opposite figure :

AD = XY = 6 cm. , D is the midpoint of $\overline{BC}$
, X is the midpoint of $\overline{BE}$, Y is the midpoint of $\overline{CE}$
Prove that : m (∠ BAC) = 90°



Quiz 4 till lesson 4 – unit 4

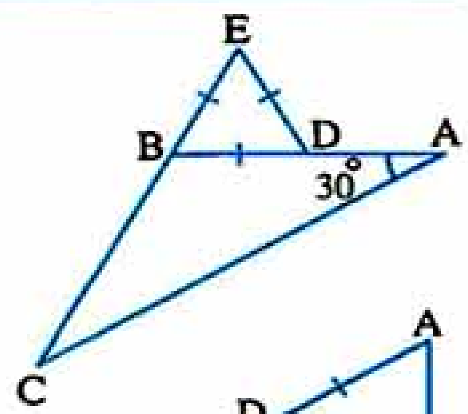
time 20 min.

1 Complete the following :

1. The isosceles triangle in which the measure of one of its angles = 60° is ……………
2. If ABC is a triangle in which : m (∠ B) = 50° and m (∠ C) = 80° , then BC = ……………
3. In Δ ABC , if m (∠ A) = 30° , m (∠ B) = 90° , then : BC = …………… AC

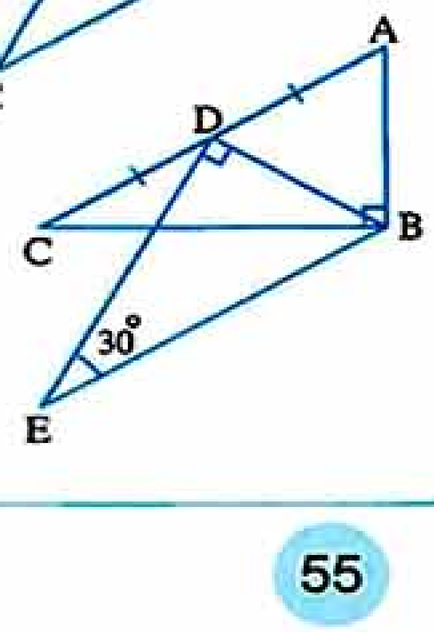
2 [a] In the opposite figure :

E ∈ $\overrightarrow{CB}$, D ∈ $\overline{AB}$,
ED = DB = EB and m (∠ A) = 30°
Prove that :
ABC is an isosceles triangle.



[b] In the opposite figure :

m (∠ ABC) = m (∠ BDE) = 90°
, m (∠ E) = 30°
, D is the midpoint of $\overline{AC}$
Prove that : AC = BE



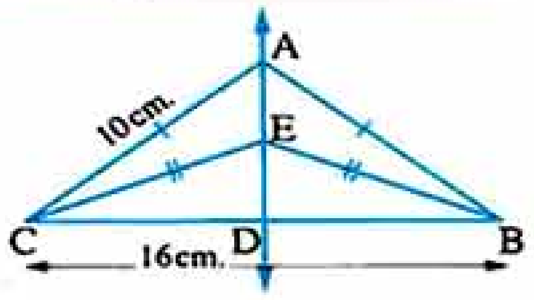
Quiz 5 till lesson 5 – unit 4

time 20 min.

1 Complete the following :

1. The bisector of the vertex angle of the isosceles triangle ....
2. If $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians , then DM = ... AD
3. Any point on the axis of symmetry of a line segment is .... from its terminals.

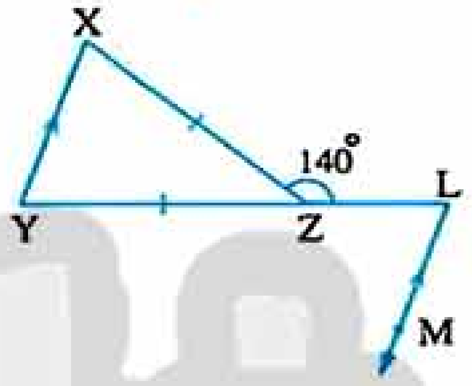
2 [a] In the opposite figure :



ABC is a triangle in which : AB = AC = 10 cm. , BE = EC , BC = 16 cm. and $\overleftrightarrow{AE} \cap \overline{BC} = \{D\}$

Find : The length of $\overline{AD}$ ABC is an isosceles triangle.

[b] In the opposite figure :



$Z \in \overline{LY}$, XZ = ZY

, m ($\angle$ LZX) = 140°

, $\overrightarrow{LM}$ // $\overline{XY}$

Find : m ($\angle$ MLY)

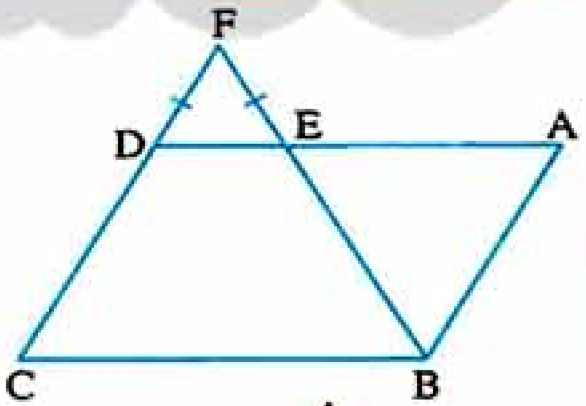
Quiz 6 till lesson 1 – unit 5

time 20 min.

1 Complete the following :

1. The measure of any exterior angle of a triangle is greater than ....
2. In Δ ABC if $\overline{AD}$ is a median , M is the point of intersection of medians , then AM = ... AD
3. If $x > y$, $z < y$, then x .... z

2 [a] In the opposite figure :



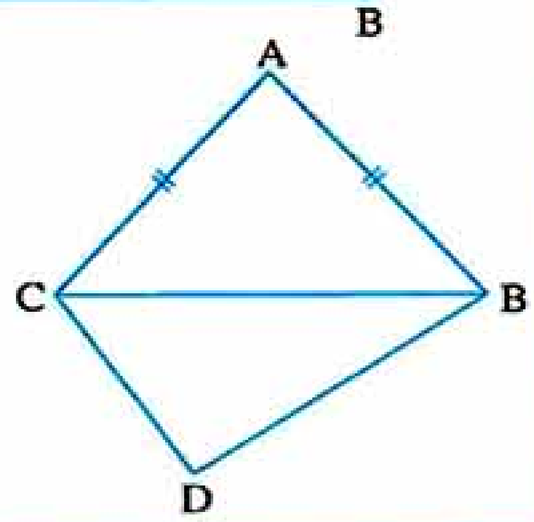
ABCD is a parallelogram ,

$E \in \overline{AD}$, $\overrightarrow{BE} \cap \overrightarrow{CD} = \{F\}$

in which EF = DF

Prove that : Δ BAE is an isosceles triangle.

[b] In the opposite figure :



AB = AC and m ($\angle$ BCD) > m ($\angle$ CBD)

Prove that :

m ($\angle$ ACD) > m ($\angle$ ABD)

Quiz 7 till lesson 2 – unit 5

time 20 min.

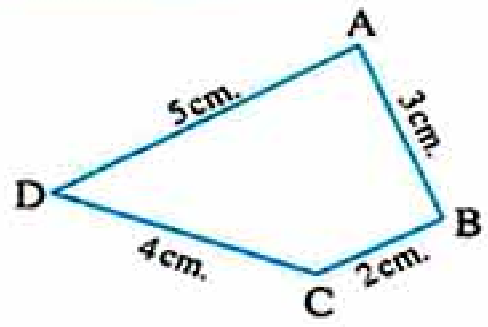
1 Complete the following :

1. In a triangle , if two sides have unequal lengths , the longer is opposite ....
2. The perpendicular to a line segment from its midpoint is .... to it.
3. If ABC is a triangle in which : AB = 4 cm. , BC = 5 cm. and AC = 6 cm. , then : m (∠ ....) > m (∠ ....) > m (∠ ....)

2 [a] In the opposite figure :

ABCD is a quadrilateral

Prove that : m (∠ ABC) > m (∠ ADC)



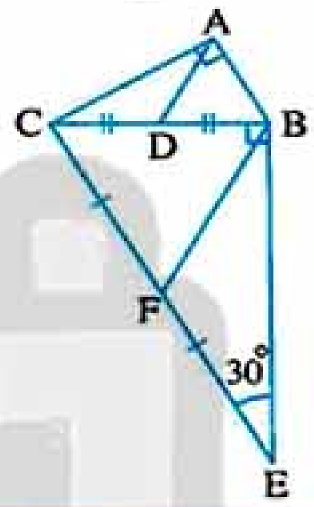
[b] In the opposite figure :

m (∠ BAC) = m (∠ CBE) = 90°

, m (∠ BEC) = 30°

, D and F are the midpoints of $\overline{BC}$ and $\overline{CE}$ respectively.

Prove that : AD = $\frac{1}{2}$ BF



Quiz 8 till lesson 3 – unit 5

time 20 min.

1 Complete the following :

1. The longest side in the right-angled triangle is ....
2. In Δ ABC : If m (∠ A) = 60° and m (∠ B) = 70° , then the shortest side is ....
3. In Δ ABC , if AB = AC , m (∠ A) = 2 m (∠ B) , then m (∠ C) = ...°

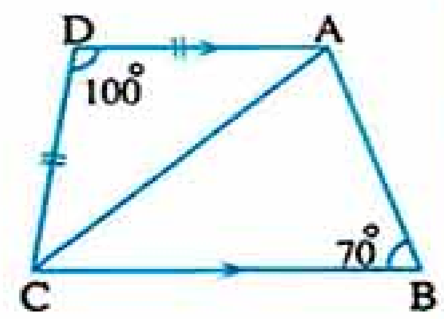
2 [a] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, AD = DC ,

m (∠ B) = 70° and m (∠ D) = 100°

Prove that :

1. AC > AB
2. Δ ABC is an isosceles triangle.



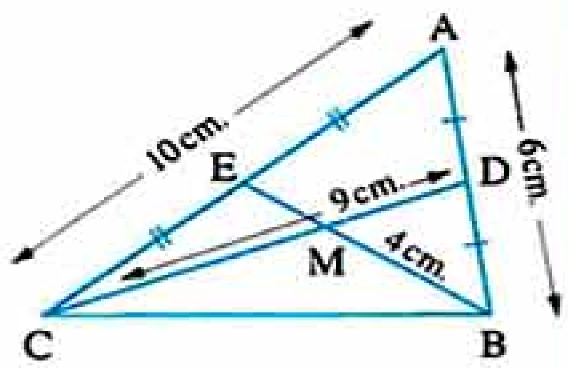
[b] In the opposite figure :

AB = 6 cm. , AC = 10 cm.

, BM = 4 cm. , CD = 9 cm.

, D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

Find : The perimeter of the figure ADME



Quiz 9 till lesson 4 – unit 5

time 20 min.

1 Choose the correct answer from the given ones :

1 In ΔABC : If $AB = 6$ cm. and $AC = 7$ cm. then $BC \in$ ....

(a) $]6 , 13]$ (b) $[6 , 7]$ (c) $]1 , 13[$ (d) $[1 , 7[$

2 An isosceles triangle in which the measure of the vertex angle is $100°$, then the measure of one of the two base angles = ...

(a) $80°$ (b) $40°$ (c) $50°$ (d) $100°$

3 The numbers that can be lengths of sides of a triangle are ....

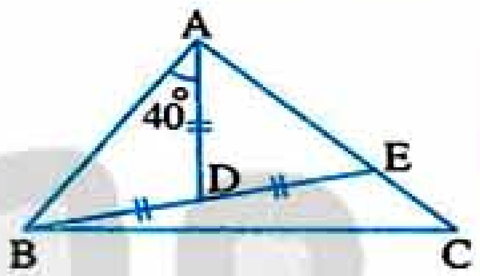
(a) 7 , 7 , 14 (b) 3 , 4 , 9 (c) 4 , 5 , 12 (d) 5 , 5 , 5

2 [a] In the opposite figure :

$AD = BD = ED$, $m(\angle DAB) = 40°$

Prove that :

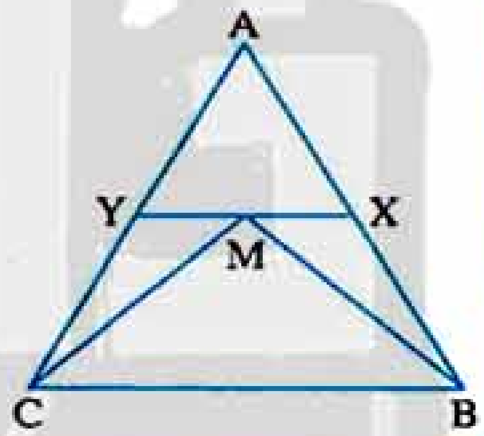
1 $AD < AB$ 2 $BC > AC$



[b] In the opposite figure :

ABC is a triangle in which $X \in \overline{AB}$, $Y \in \overline{AC}$, $M \in \overline{XY}$

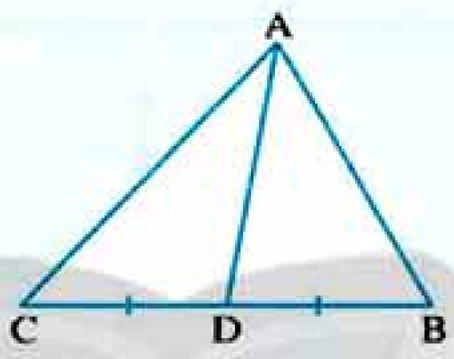
Prove that : $AB + AC > MB + MC$



Revision for the important theorems , corollaries and rules of geometry

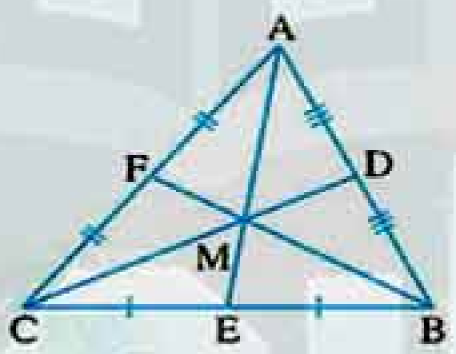
Medians of triangle

The median of the triangle is the line segment drawn from any vertex of the triangle vertices to the midpoint of the opposite side of this vertex.



If D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is a median in ΔABC

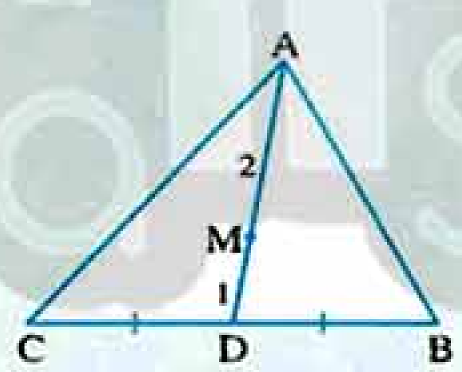
The medians of a triangle are concurrent.



If $\overline{CD}$, $\overline{BF}$ and $\overline{AE}$ are the medians of ΔABC where $\overline{CD} \cap \overline{BF} \cap \overline{AE} = \{M\}$, then M is the intersection point of medians of ΔABC

The point of concurrence of the medians of the triangle divides each median in the ratio of :

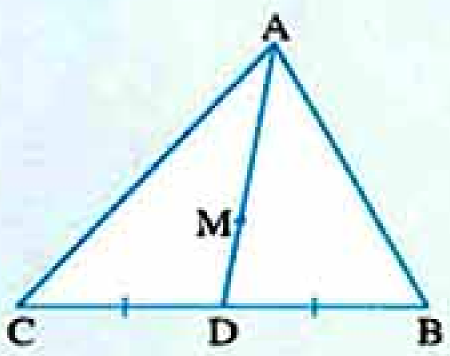
- 1 : 2 from the base.
- 2 : 1 from the vertex.



If M is the intersection point of medians of ΔABC, then :

- $DM = \frac{1}{2} AM$
- $AM = 2\,DM$
- $DM = \frac{1}{3} AD$
- $AM = \frac{2}{3} AD$

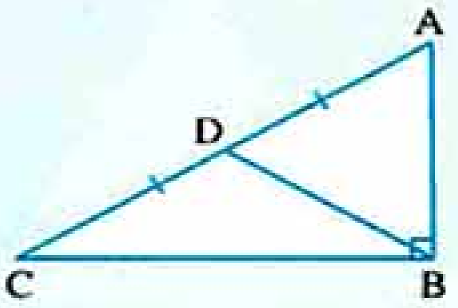
The point which divides the median in a triangle by the ratio 1 : 2 from the base is the point of the intersection of the medians of the triangle.



If DM : MA = 1 : 2, then M is the intersection point of medians of ΔABC

Right-angled triangle

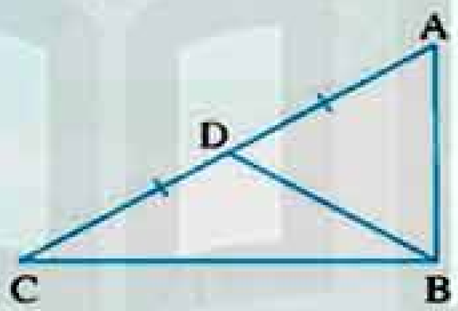
The length of the median from the vertex of the right angle equals half the length of the hypotenuse.



If Δ ABC is right-angled at B , $\overline{BD}$ is a median in it , then

$BD = \frac{1}{2} AC$

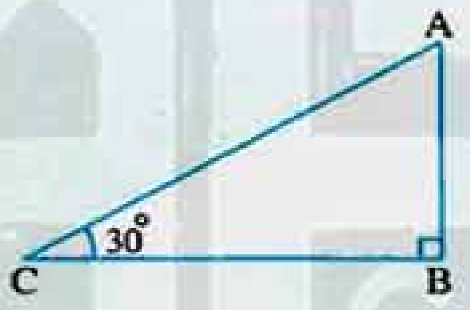
If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.



If $\overline{BD}$ is a median in Δ ABC , $BD = \frac{1}{2} AC$

$\therefore m(\angle ABC) = 90°$

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

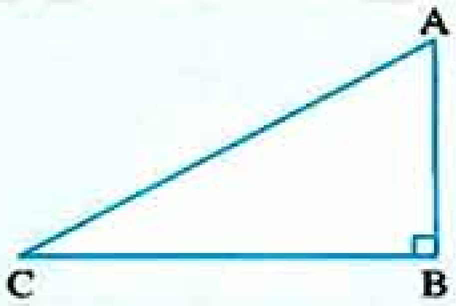


If Δ ABC is a right-angled at B in which :

$m(\angle C) = 30°$

, then $AB = \frac{1}{2} AC$

In the right-angled triangle, the hypotenuse is the longest side of the triangle.

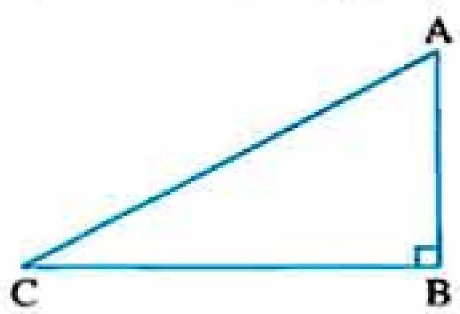


If Δ ABC is a right-angled at B , then

$AC > AB$, $AC > BC$

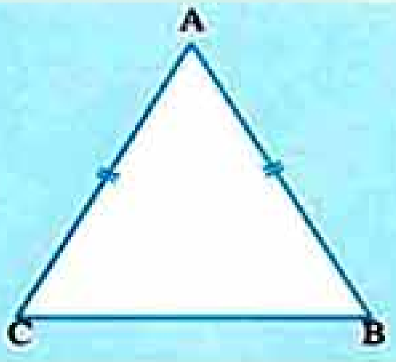
If Δ ABC is a right-angled at B , then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



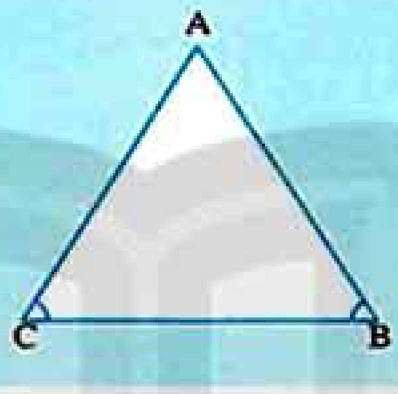
The isosceles triangle

The base angles of the isosceles triangle are congruent.



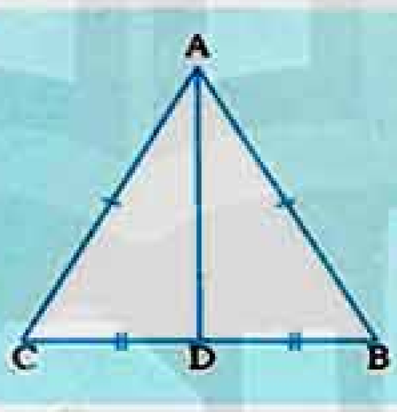
If ΔABC in which :
$AB = AC$, then
$m(\angle B) = m(\angle C)$

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



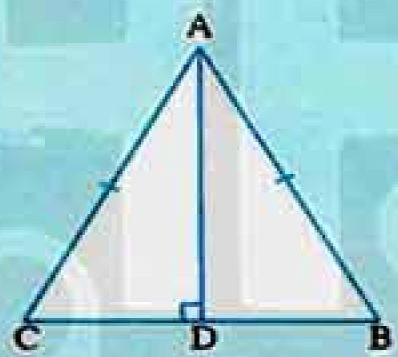
If ΔABC in which :
$m(\angle B) = m(\angle C)$
, then $AB = AC$

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.



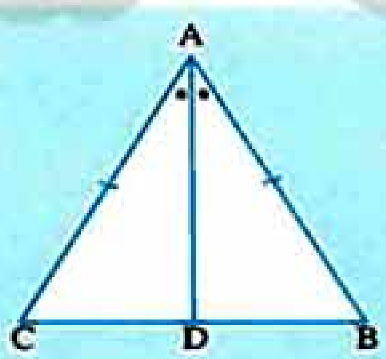
If ΔABC in which :
$AB = AC$, $\overline{AD}$ is a median
, then $\overrightarrow{AD}$ bisects $\angle BAC$
, $\overline{AD} \perp \overline{BC}$

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



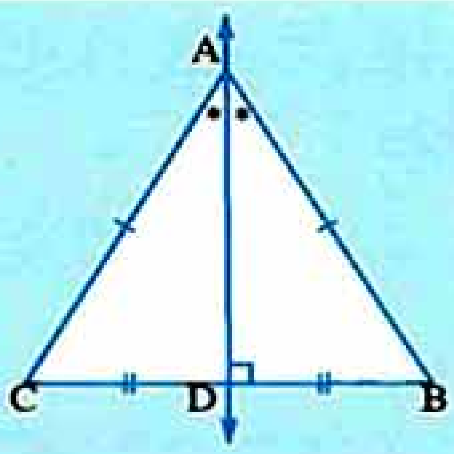
If ΔABC in which :
$AB = AC$, $\overline{AD} \perp \overline{BC}$
, then D is the midpoint of $\overline{BC}$,
$\overrightarrow{AD}$ bisects $\angle BAC$

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.



If ΔABC in which :
$AB = AC$, $\overrightarrow{AD}$ bisects $\angle BAC$, then D is the midpoint of $\overline{BC}$, $\overline{AD} \perp \overline{BC}$

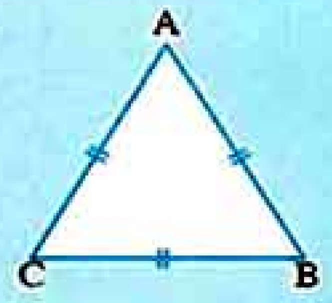
The number of axes of symmetry of the isosceles triangle = 1



If ΔABC in which :
$AB = AC$, $\overleftrightarrow{AD} \perp \overline{BC}$ and intersect it at D
, then $\overleftrightarrow{AD}$ is the axis of symmetry of the triangle ABC

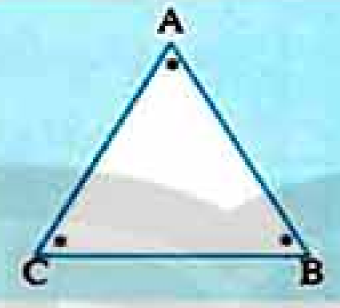
The equilateral triangle

If the triangle is an equilateral , then it is equiangular where each angle measure is 60°



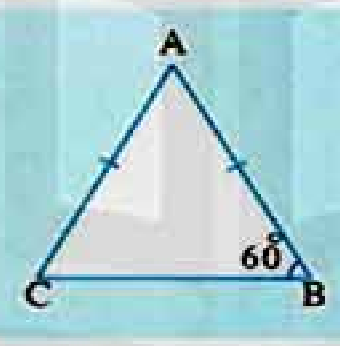
If Δ ABC in which :
AB = BC = CA , then
$m(\angle A) = m(\angle B) = m(\angle C) = 60°$

If the angles of a triangle are congruent , then the triangle is equilateral.



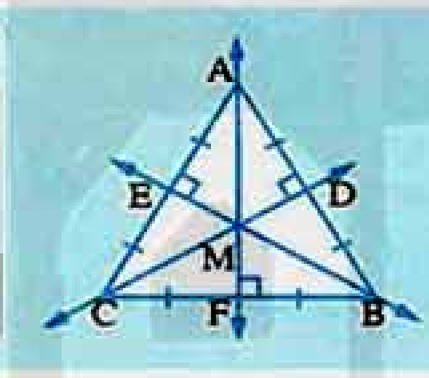
If Δ ABC in which :
$m(\angle A) = m(\angle B) = m(\angle C)$
, then AB = BC = CA

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.



If Δ ABC in which :
AB = AC , $m(\angle B) = 60°$
, then Δ ABC is an equilateral triangle.

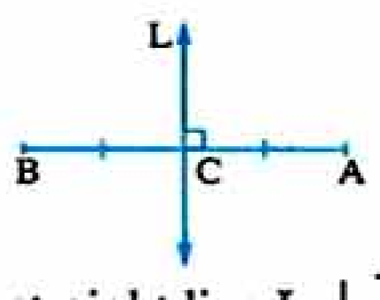
The equilateral triangle has three axes of symmetry.



If Δ ABC is an equilateral triangle
, $\overleftrightarrow{AF} \perp \overline{BC}$, $\overleftrightarrow{CD} \perp \overline{AB}$, $\overleftrightarrow{BE} \perp \overline{AC}$
, then $\overleftrightarrow{AF}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{BE}$ are the axes of symmetry of the triangle ABC

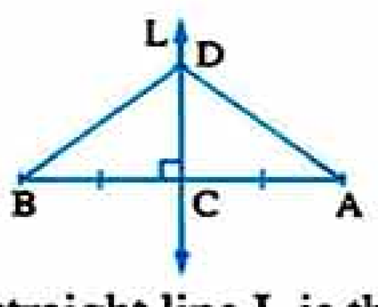
The axis of symmetry

The axis of symmetry of a line segment is the straight line perpendicular to it from its middle.



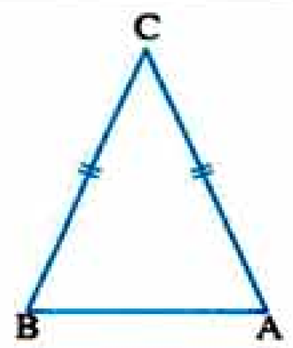
If the straight line L $\perp \overline{AB}$, C $\in \overline{AB}$ where CA = CB , C $\in$ the straight line L , then L is the axis of $\overline{AB}$

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).



If the straight line L is the axis of $\overline{AB}$, D $\in$ the straight line L , then DA = DB

If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

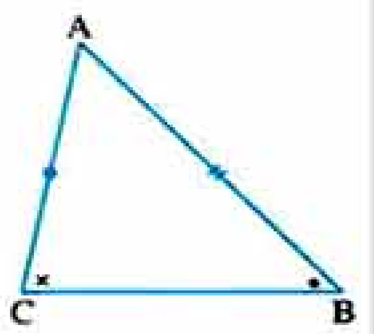


If CA = CB , then C lies on the axis of $\overline{AB}$

Inequality relations in the triangle

Comparing the measures of angles in a triangle

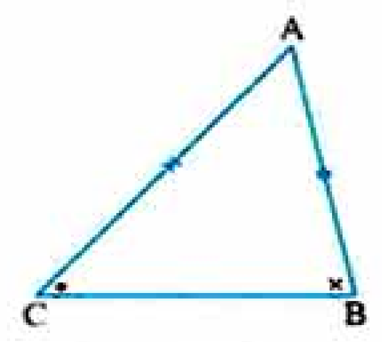
If two sides have unequal lengths , the longer is opposite to the angle of the greater measure



If AB > AC , then m (∠ C) > m (∠ B)

Comparing the lengths of sides in a triangle

If two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



If m (∠ B) > m (∠ C) , then AC > AB

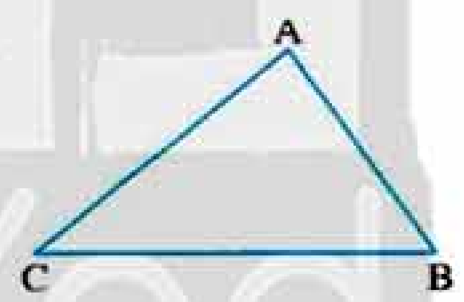
Triangle inequality

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

AB + BC > AC

, BC + CA > AB

, CA + AB > BC

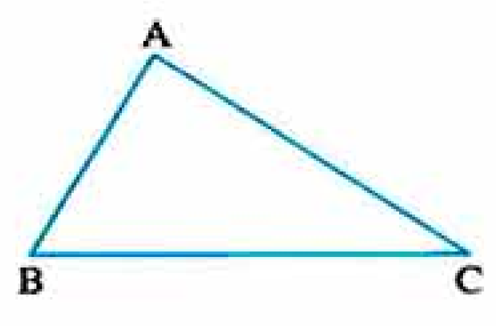


Notice that

- The length of any side in a triangle is greater than the difference between the lengths of the two other sides and less than their sum.

  In Δ ABC :

  AC – AB < BC < AC + AB

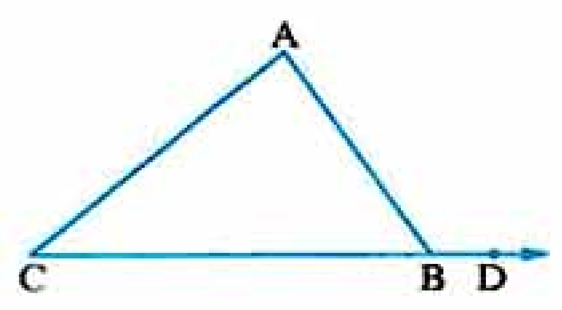


- The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

  In Δ ABC :

  m (∠ ABD) > m (∠ A)

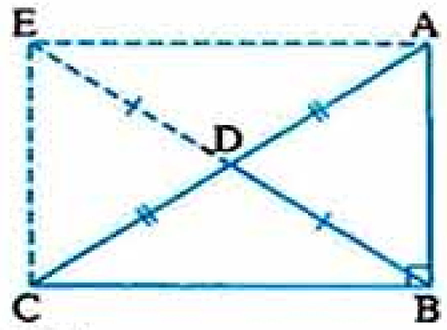
  , m (∠ ABD) > m (∠ C)



Proofs of the important theorems

Theorem

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.



Given ABC is a triangle in which m (∠ ABC) = 90° ,

$\overline{BD}$ is a median in the triangle ABC

R.T.P. $BD = \frac{1}{2} AC$

Construction Draw $\overrightarrow{BD}$ and take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof In the figure ABCE : ∵ $\overline{AC}$ and $\overline{BE}$ bisect each other

∴ The figure ABCE is a parallelogram.

∵ m (∠ ABC) = 90°

∴ The figure ABCE is a rectangle.

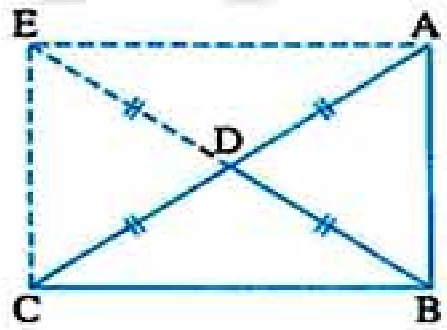
∴ BE = AC

, ∵ $BD = \frac{1}{2} BE$

∴ $BD = \frac{1}{2} AC$ (Q.E.D.)

Theorem

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.



Given In Δ ABC , $\overline{BD}$ is a median and DA = DB = DC

R.T.P. m (∠ ABC) = 90°

Construction Draw $\overrightarrow{BD}$, then take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof ∵ $BD = \frac{1}{2} BE = \frac{1}{2} AC$ ∴ BE = AC

∴ In the figure ABCE :

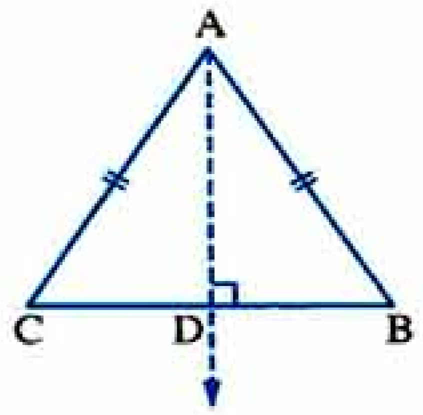
$\overline{AC}$ and $\overline{BE}$ are equal in length and bisect each other.

∴ The figure ABCE is a rectangle.

∴ m (∠ ABC) = 90° (Q.E.D.)

Theorem

The base angles of the isosceles triangle are congruent.



Given ABC is a triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P. $\angle B \equiv \angle C$

Construction Draw $\overrightarrow{AD} \perp \overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$

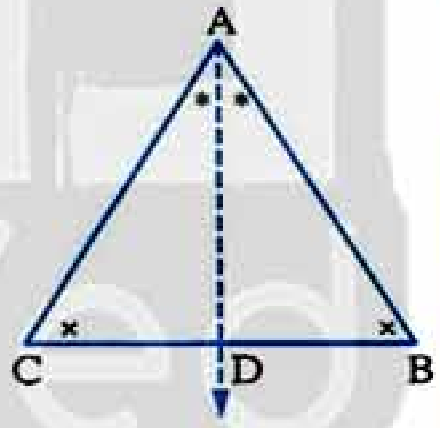
Proof $\because \Delta\Delta$ ADB , ADC in which :

$$\begin{cases} m(\angle ADB) = m(\angle ADC) = 90^\circ & \text{(const.)} \\ \overline{AB} \equiv \overline{AC} & \text{(given)} \\ \overline{AD} \text{ is a common side} & \end{cases}$$

$\therefore \Delta ADB \equiv \Delta ADC$, then we deduce that $\angle B \equiv \angle C$ (Q.E.D.)

Theorem

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



Given ΔABC in which $\angle B \equiv \angle C$

R.T.P. $\overline{AB} \equiv \overline{AC}$

Construction bisect $\angle BAC$ by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D

Proof $\because \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\because \overrightarrow{AD}$ bisects $\angle BAC$ $\therefore m(\angle BAD) = m(\angle CAD)$

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore m(\angle ADB) = m(\angle ADC)$

$\therefore$ In $\Delta\Delta$ ABD and ACD :

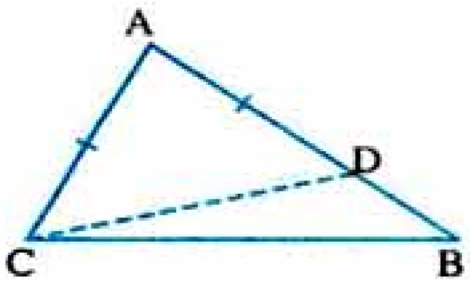
$$\begin{cases} \overline{AD} \text{ is a common side} \\ m(\angle BAD) = m(\angle CAD) \text{ (const.)} \\ m(\angle ADB) = m(\angle ADC) \text{ (by proof)} \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that

$\overline{AB} \equiv \overline{AC}$, then ΔABC is an isosceles triangle. (Q.E.D.)

Theorem

In a triangle , if two sides have unequal lengths , the longer is opposite to the angle of the greater measure.



Given ABC is a triangle in which AB > AC

R.T.P. m (∠ ACB) > m (∠ ABC)

Construction Take D ∈ $\overline{AB}$ such that AD = AC

Proof In Δ ACD : ∵ AD = AC ∴ m (∠ ADC) = m (∠ ACD) (1)

∵ ∠ ADC is an exterior angle of Δ DBC

∴ m (∠ ADC) > m (∠ B) (2)

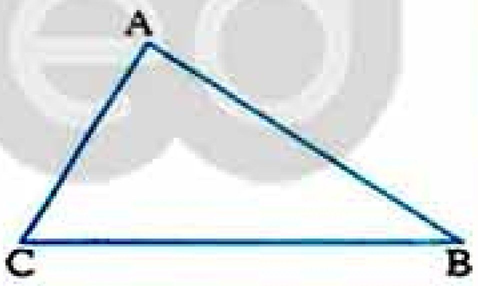
From (1) and (2) : ∴ m (∠ ACD) > m (∠ B)

, ∵ m (∠ ACB) > m (∠ ACD)

∴ m (∠ ACB) > m (∠ ABC) (Q.E.D.)

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



Given ABC is a triangle in which m (∠ C) > m (∠ B)

R.T.P. AB > AC

Proof ∵ $\overline{AB}$ and $\overline{AC}$ are two line segments.

∴ One of the following cases should be verified.

1. AB > AC 2. AB = AC 3. AB < AC

Unless AB > AC , then either AB = AC **or** AB < AC

- If : AB = AC , then m (∠ C) = m (∠ B) and this contradicts the given where m (∠ C) > m (∠ B)
- If : AB < AC , then m (∠ C) < m (∠ B) according to the preceding theorem.

Again this contradicts the given , where m (∠ C) > m (∠ B)

∴ It should be that AB > AC (Q.E.D.)